Read each problem carefully. Show all work for each problem. Electronic devices and notes *not* allowed.

1. **(12pts)** Consider an object moving with acceleration $\boldsymbol{a}(t) = \langle 2,\ 4\sin(2t)\,,\ 6t\rangle$, with initial velocity and position given by $\boldsymbol{v}(0) = \langle 0,\ 0,\ 0\rangle$ and $\boldsymbol{r}(0) = \langle 1,\ 0,\ 2\rangle$.

a) **(6pts)** Find the velocity vector, $\boldsymbol{v}(\boldsymbol{t})$.

b) **(6pts)** Find the position vector, $\boldsymbol{r}(\boldsymbol{t})$.

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2. **(13pts)** For the function $f(x,y) = \ln(4 - x^2 - 4y^2)$

a) **(5pts)** Find and sketch the domain as a region in the (*x*, *y*) plane. Is it an open or a closed region?

b) **(3pts)** Determine the range of $f(x,y)$.

c) **(5pts)** Find the equations of the level curves corresponding to $f = 0$ and $f = 1$. Make a *rough* sketch of these level curves on the same coordinate axes.

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3. **(12pts)** Consider the following two limits. If the limit exists, then find its value. If it does not, then prove this using a two-path test:

$$(a)\quad \lim_{(x,y)\to(1,2)} \frac{x^2 + xy - 3x}{x + y - 3} \qquad (b)\quad \lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + xy + y^2}$$

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4. **(12pts)** Consider the surface $2x^2 + 3y^2 + 2z^2 + 2xz = 9$.

a) **(6pts)** Find the tangent plane to this surface at point $P(1,1,1)$.

b) **(6pts)** Use linearization to estimate z on this surface near $P(1,1,1)$ when $x = 1.02$, $y = 0.99$.

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5. **(12pts)** Consider the function $f(x,y,z) = xe^y + yz^2$ and points $P(2,0,1)$ and $Q(4,-1,3)$.

a) **(4pts)** Find the gradient of function $f(x,y,z)$ at point P.

b) **(4pts)** Find the maximum rate of change of $f(x,y,z)$ at P, and the unit vector in the direction of this maximum rate of change.

c) **(4pts)** Find the directional derivative of $f(x,y,z)$ in the direction from point P toward point Q.

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6. **(13pts)** Find all critical points of $f(x,y) = x^3 + y^3 - 3xy$, and classify each as a local maximum, local minimum, or saddle point using the second derivative test.

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7. **(13pts)** Use Lagrange multipliers to find the maximum and minimum *values* of $f(x,y,z) = 2x - y + 2z$ on the surface of the sphere $x^2 + y^2 + z^2 = 9$. State the points at which such maximum and minimum values are achieved.

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8. **(13pts)** Find the second-degree Taylor polynomial for $f(x,y) = e^x \cos y$ centered at (0,0). Use your polynomial to approximate $f(0.1, 0.2)$.