Math 340 • Fall 2026 • Homework #0 • Victor Matveev

This assignment checks basic Calculus pre-requisites for this course. Although it will not be collected, you should expect a quiz based on this material at the beginning of the semester.

1) Consider the linear approximation $\sqrt{1+x} \approx 1 + \frac{x}{2}$

a) Use it to estimate $\sqrt{5}$ without a calculator. Hint: write $5 = 4 + 1 = 4\left(1 + \frac{1}{4}\right)$.

b) Compare your approximation in part "a" with the exact value of $\sqrt{5}$.

2) Consider the function $f(x) = x^{1/3}$:

a) Find the quadratic Taylor polynomial of $f(x) = x^{1/3}$ centered around $x_0 = 1$

b) Use your quadratic Taylor polynomial to estimate $f(0.5)$.

c) Compute the absolute error of your estimate by comparing it with the exact value $f(0.5) = 2^{-1/3}$.

d) Use the Taylor remainder formula to find the upper bound for the approximation error. Compare this bound with the actual error found in part "c".

3) Find the quadratic Taylor polynomial for $f(x) = e^{\sin(x)}$ centered at $x_0 = 0$ by substituting the Taylor series for $\sin(x)$ into the Taylor series for $\exp(u)$ (see below), keeping only terms up to degree 2:

- $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$
- $\exp(u) = 1 + u + \frac{u^2}{2} + \frac{u^3}{3!} + \cdots$

Check you Taylor polynomial by comparing its value with the exact value of $f(x)$ at $x = 0.2$.

4) Find the seventh derivative $f^{(7)}(0)$ of function $f(x) = x\exp(-x^2)$ without repeated differentiation of $f(x)$.

Hint: use the Taylor expansion of $\exp(u)$ in the previous problem (with $u = -x^2$) , and identify the coefficient of term x^7 in the result.

5) Consider the function $f(x) = \frac{1}{x}$. Given $a = -1$ and $b = 1$, we obtain:

$$\frac{f(b) - f(a)}{b - a} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{1 - (-1)}{2} = 1$$

Does the Mean Value Theorem guarantee that there exists $c \in (-1, 1)$ where $f'(c) = 1$? Explain.