Math 340 • Fall 2026 • Homework #1 • Victor Matveev

1. **(10pts)** Consider the linear approximation $\sqrt[3]{1+x} \approx 1 + \frac{x}{3}$

   a) Use it to estimate $\sqrt[3]{6}$ without a calculator. Hint: write $6 = 8 - 2 = 8\left(1 - \frac{1}{4}\right)$.

   b) Compare your approximation in part "a" with the exact value of $\sqrt[3]{6}$.

2. **(35pts)** Consider the function $f(x) = \ln(x)$:

   a) Find the quadratic Taylor polynomial of $f(x)$ centered around $a = 1$. Start by computing the relevant derivatives of function $f(x)$ at $a = 1$.

   b) Use your quadratic Taylor polynomial to estimate $f(0.8)$.

   c) Find an upper bound on the **remainder** of this quadratic polynomial approximation. Compare it with the **actual error** of your polynomial estimate of $f(0.8)$.

3. **(20pts)** Find the derivative $f^{(5)}(0)$ of function $f(x) = x\cos(2x)$ without repeated differentiation of $f(x)$.

   Hint: use the Taylor expansion of $\cos(u)$ with $u = 2x$, and identify the coefficient of term x^5 in the result.

4. **(35pts)** Find the 4th degree Taylor polynomial for $f(x) = \sqrt{\cos x}$ centered at $x_0 = 0$ by combining the Taylor series for $\cos(x)$ with the series for $\sqrt{1+x}$ (see below), keeping only terms up to degree 4.

   Check you Taylor polynomial by comparing its value with the exact value of $f(x)$ at $x = 0.2$.

In your solution (except problem 2), you may use the following standard expansions:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + O(x^7) \qquad \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + O(x^6) \qquad \exp(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + O(x^4)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + O(x^5) \quad \Rightarrow \quad \ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + O(x^5)$$

$$(1\pm x)^p = 1 \pm px + \frac{p(p-1)}{2}x^2 + O(x^3) \Rightarrow \begin{cases} p = \frac{1}{2}: & \sqrt{1\pm x} = 1 \pm \frac{x}{2} - \frac{x^2}{8} + O(x^3) \\ p = -\frac{1}{2}: & \frac{1}{\sqrt{1\pm x}} = 1 \mp \frac{x}{2} + \frac{3x^2}{8} + O(x^3) \\ p = -1: & \frac{1}{1\pm x} = 1 \mp x + x^2 \mp x^3 + O(x^4) \end{cases}$$