Math 340 • Fall 2026 • Homework #3 • Victor Matveev

Sections 1.1-1.2: Bisection method and the direct iteration (fixed-point Iteration) method

Please show all your work.

Problem 1 (40pts) Consider the root (zero) of function $f(x) = x^3 - x - 1$.

a) Find an obvious bracketing interval $[a, b]$ (with integer *a* and *b*) that contains the positive root of $f(x)$. Make a plot of $f(x)$ over this interval.

b) Perform three iterations of the bisection method, "manually".

c) Find the number of bisection steps required to guarantee an absolute error of 10^{-9} or smaller (count the initial calculation $x_1 \equiv c_1 = \frac{a+b}{2}$ as the first iteration).

d) Transform the root-finding problem $\boldsymbol{f(x) = 0}$ to a fixed-point problem $\boldsymbol{x = g(x) \equiv x^3 - 1}$. Perform two iterations $x_{n+1} = g(x_n)$, starting with an initial guess $x_0 = 1$. Does this iteration converge? Explain.

Problem 2 (60pts) Consider the following two functions $g(x)$:

$$(a)\ g(x) = x^2 + \frac{3}{2}(1-x) \qquad (b)\ g(x) = \frac{3}{4}\left(x + \frac{1}{x}\right)$$

For each of these two functions, do the following:

i. Find all fixed points of the iteration $\boldsymbol{x_{n+1} = g(x_n)}$ (i.e. solve the equation $r = g(r)$).

ii. For each of the fixed points *r*, indicate whether the iteration would converge (at least locally). In other words, determine which of the fixed points are stable. To do that, determine whether the stability condition (contraction property) is satisfied:

$$C = |g'(r)| < 1$$

iii. For each stable fixed point, estimate the number of steps needed to achieve an absolute root error (tolerance) of 10^{-13}, assuming that the initial guess has an error of about 0.1.

Hint: $\epsilon_n \approx C^n \epsilon_0$

Review of bisection and direction iteration is here:
https://web.njit.edu/~matveev/Courses/M340_F26/M340_F26_Bisection_Iteration.pdf