Math 340 • Fall 2026 • Homework #4 • Victor Matveev

Sections 1.4-1.5: Newton's method and the Secant method.

Please show all your work. **Lecture notes URL for this material is: [PDF]**

Hints / reminders: Newton's method iteration: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$.

Secant method iteration: $x_{n+1} = x_n - f(x_n)\dfrac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$. Note: two points needed!

Problem 1 (40pts) Consider the problem of finding the root (zero) of the function $\boldsymbol{f(x) = x - \mathbf{e}^{-x}}$.

a) Set up the Newton's iterations for finding the root of $f(x)$, and compute x_1 and x_2, starting with $x_0 = 0$ (write down the **exact** expression for x_2, along with its decimal approximation to 6 digits).

b) Set up and perform **one** iteration of the Secant method to find x_2, starting with $x_0 = 0$ and $x_1 = 1$ (recall that two starting values are required). What happens if you start with $x_0 = 1$ and $x_1 = 0$ instead?

c) Which of the two values x_2 found in (a) and (b) is more accurate? Answer by comparing the so-called "residuals" for each method, $f(x_2)$ (i.e. compare how close the function value is to zero at x_2)

Problem 2 (60pts) Consider the problem of finding the principal value of $\boldsymbol{\arctan(A)}$, which is equivalent to the problem for finding the principal root of function $\boldsymbol{f(x) = \tan(x) - A}$, contained within interval $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

a) Set up the Newton's iteration for finding the root of the function $\boldsymbol{f(x) = \tan(x) - A}$.

b) For $\boldsymbol{A = 2}$, perform 1 iteration of the Newton's method to find x_1, starting with $x_0 = 1$.

c) Compute the absolute errors of your iterates x_0 and x_1 by comparing them to the true root:

$$\arctan 2 = 1.1071487177.$$

d) For Newton's method, the asymptotic error (i.e. error close to the root) satisfies

$$\epsilon_{n+1} \approx M\epsilon_n^2 \quad \text{where} \quad M = \frac{f''(r)}{2f'(r)}.$$

Given $\boldsymbol{f(x) = \tan(x) - A}$, simplify the above expression for M. For $A = 2$, compare $\epsilon_1 \approx M\epsilon_0^2$ with the actual error ϵ_1 found in part "c". How accurate is this approximation for the error?

e) Use your result in part "d" to estimate the number of steps required to reach the absolute tolerance $\epsilon_n \leq 2^{-53} \approx 10^{-16}$, given that $\epsilon_0 = 2^{-3} = 0.125$.