Math 340 • Fall 2026 • Homework #7 • Victor Matveev

Sections 3.1: Polynomial interpolation.

Please show all your work.

See pages 3-4 of lecture notes for an example with four points / cubit interpolation:
https://web.njit.edu/~matveev/Courses/M340_F25/M340_F25_Interpolation.pdf

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Problem 1 (50pts): Polynomial interpolation (Lagrange's method vs. Newton's divided differences)

Consider polynomial interpolation for the following data set:

$$\{(x_k, y_k)\} = \{(0,3), (1,6), (3,0)\}$$

Note: The three x-values are **not** equally spaced.

a) Find the **quadratic interpolating polynomial** using the **Lagrange basis.**

b) Find the **quadratic interpolating polynomial** using **Newton's divided differences.**

c) Show that the two polynomials are **identical** by expressing both in the **monomial form** (i.e., as $ax^2 + bx + c$)

d) **Plot** the interpolating polynomial together with the given data points over the interval [0, 3].

Problem 2 (50pts): Interpolation vs. Taylor Approximation

Find the **quadratic interpolating polynomial** $P_2(x)$ for

$$f(x) = \frac{1}{1+x}$$

using its values at

$$\{x_k\} = \left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$$

You can use *any* method to construct it. Simplify and express your result in monomial form (i.e. as $ax^2 + bx + c$).

Then, compare this interpolating polynomial with the **quadratic Taylor polynomial**

$$T_2(x) = 1 - x + x^2.$$

Finally, **plot** on the same graph $f(x)$, $P_2(x)$ and $T_2(x)$, all on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ using MATLAB or another computational tool.

Based on your plot, briefly compare the accuracy of the two approximations near $x = 0$ and near the endpoints of the interpolation interval.