Math 340 • Fall 2026 • Homework #7 • Victor Matveev

Sections 3.1: Polynomial interpolation.

Please show all your work.

Problem 1 (50pts): Polynomial interpolation (Lagrange's method vs. Newton's divided differences)

Consider polynomial interpolation for the following data set:

$$\{(x_k, y_k)\} = \{(0,3), (1,6), (3,0)\}$$

Note: The three x-values are **not** equally spaced.

a) Find the **quadratic interpolating polynomial** using the **Lagrange basis.**

$$f(x) \sim P(x) = 3\cdot L_1(x) + 6\cdot L_2(x) + 0\cdot L_3(x)$$

$$= 3\frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + 6\frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}$$

$$= 3\frac{(x-1)(x-3)}{(0-1)(0-3)} + 6\frac{(x-0)(x-3)}{(1-0)(1-3)}$$

$$= (x-1)(x-3) - 3x(x-3) = -(x-3)(2x+1) = \boxed{-2x^2+5x+3}$$

b) Find the **quadratic interpolating polynomial** using **Newton's divided differences.**

x_k	$f[x_k]$	$f[x_{k-1}, x_k]$	$f[x_{k-2}, x_{k-1}, x_k]$
0	3		
1	6	$\frac{6-3}{1-0} = 3$	
3	0	$\frac{0-6}{3-1} = -3$	$\frac{-3-3}{3-0} = -2$

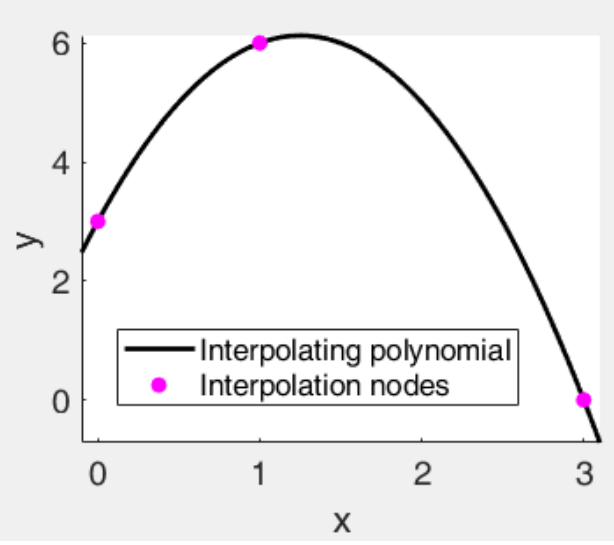
$$f(x) \sim P(x) = f[x_1] + f[x_1, x_2](x-x_1) + f[x_1, x_2, x_3](x-x_1)(x-x_2)$$

$$= 3 + 3x - 2x(x-1)$$

$$= \boxed{-2x^2+5x+3}$$

c) Show that the two polynomials are **identical** by expressing both in the **monomial form** (i.e., as $ax^2 + bx + c$)

d) **Plot** the interpolating polynomial together with the given data points over the interval [0, 3].



Problem 2 (50pts): Interpolation vs. Taylor Approximation

Find the **quadratic interpolating polynomial** $P_2(x)$ for

$$f(x) = \frac{1}{1+x}$$

using its values at

$$\{x_k\} = \left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$$

You can use *any* method to construct it. Simplify and express your result in monomial form (i.e. as $ax^2 + bx + c$).

x_k	$f[x_k] = \frac{1}{1+x_k}$	$f[x_{k-1}, x_k]$	$f[x_{k-2}, x_{k-1}, x_k]$
$-\frac{1}{2}$	2		
0	1	$\frac{1-2}{0-(-1/2)} = -2$	
$+\frac{1}{2}$	$\frac{2}{3}$	$\frac{2/3-1}{1/2-0} = -\frac{2}{3}$	$\frac{-2/3-(-2)}{1/2-(-1/2)} = \frac{4}{3}$

$$f(x) \sim P_2(x) = f[x_1] + f[x_1, x_2](x - x_1) + f[x_1, x_2, x_3](x - x_1)(x - x_2)$$

$$= 2 - 2\left(x + \frac{1}{2}\right) + \frac{4}{3}\left(x + \frac{1}{2}\right)x = \boxed{1 - \frac{4}{3}x + \frac{4}{3}x^2}$$

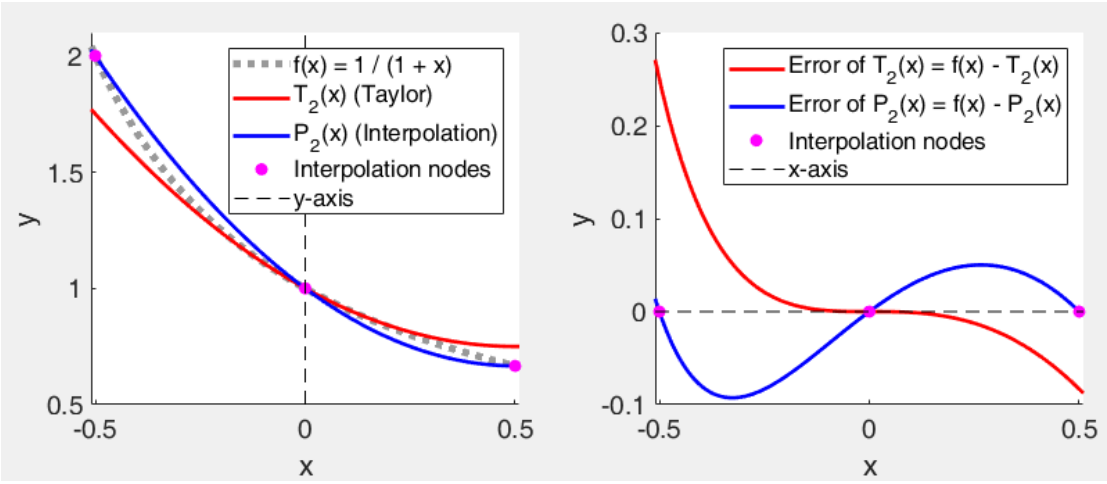
Then, compare this interpolating polynomial with the **quadratic Taylor polynomial**

$$T_2(x) = 1 - x + x^2.$$

The constant term is the same (due to interpolation node at x=0), but the linear and quadratic coefficients differ by 33%.

Finally, **plot** on the same graph $f(x)$, $P_2(x)$ and $T_2(x)$, all on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Based on your plot, briefly compare the accuracy of the two approximations near $x = 0$ and near the endpoints of the interpolation interval.



The error of the Taylor polynomial is much smaller near the origin, on the interval of about [-1/4, 1/4]

However, its error grows rapidly at the edges, so the interpolating polynomial has a smaller maximal error.