Math 340 • Fall 2026 • Homework #9 • Victor Matveev

Sections 3.4, 5.1: Quadratic Spline; Numerical Differentiation.

Problem 1 (20pts) Quadratic Spline

Find a **quadratic spline** interpolation for the data points $\{(\boldsymbol{x_k}, \boldsymbol{y_k})\} = \{(\mathbf{0},\mathbf{0}), (\mathbf{1},\mathbf{2}), (\mathbf{2},\mathbf{3}), (\mathbf{3},\mathbf{1})\}$.

As an extra condition, require extra smoothness at node x_2: $s_1''(x_2) = s_2''(x_2)$.

Reminder: the quadratic spline will have the form
$$\begin{cases} s_1(x) = y_1 + a_1(x-x_1)^2 + b_1(x-x_1), & x \in [x_1, x_2] \\ s_2(x) = y_2 + a_2(x-x_2)^2 + b_2(x-x_2), & x \in [x_2, x_3] \\ s_3(x) = y_3 + a_3(x-x_3)^2 + b_3(x-x_3), & x \in [x_3, x_4] \end{cases}$$

Since there are **6** unknowns, we need **6** constraints:

- Continuity of the spline (**3** constraints), i.e. correct values at right endpoints: $\boldsymbol{s_k(x_{k+1}) = y_{k+1}}$
- Continuity of spline derivative at all internal nodes (**2** constraints): $\boldsymbol{s_k'(x_{k+1}) = s_{k+1}'(x_{k+1})}$
- **One** extra constraint is needed: impose continuity of 2nd derivative at the 1st internal node: $\boldsymbol{s_1''(x_2) = s_2''(x_2)}$.

Finally, **plot** on separate graphs the spline, its derivative, and its 2nd derivative (by hand is fine).

Hint: first, consider equations for the coefficients of first 2 parts of the spline, $s_1(x)$ and $s_2(x)$. This will immediately reduce to 3 equations for 3 unknowns. After solving this linear system, find the coefficients of the final 3rd piece, $s_3(x)$.

Note: in class, as an extra condition we imposed $s_1''(x_1) = 0$, which is a slightly simpler case, as compared to this problem.

Problem 2 (40pts) Numerical Differentiation

Consider the following 3-point finite difference approximation of the 1st derivative of smooth function $f(x)$:

$$f'(x) = \frac{-f(x+4h) + 4f(x+2h) - 3f(x)}{4h} + \text{Error}(h)$$

a) Use Taylor expansion to find the error in the expression above. Is this method first- or second-order accurate?

b) Check your error expression by applying this finite difference approximation to the function $f(x) = x^3$.

Hint: in part "**b**", you will need a binomial expansion $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

Problem 3 (40pts) Differentiation

Use functions values $\boldsymbol{f(x-3h)}, \boldsymbol{f(x)}, \text{and } \boldsymbol{f(x+2h)}$ to find the finite difference for $\boldsymbol{f''(x)}$.

To do this, set up and solve the linear system of equations for constants (a, b, c) that results from expanding

$$af(x-3h) + b\,f(x) + c\,f(x+2h)$$

using a Taylor series around x up to 3rd order in *h*.

Don't forget to obtain the truncation error, as well. Is the expression you obtained first- or second-order accurate?

Hint: we did a similar example in class; see lecture notes:

https://mvvik.github.io/Math340_F26/M340_F26_Differentiation.pdf